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By Hsü Chang-pen

- COMMUNIST CHINA -

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GENERAL BOUNDARY RELATIONS FOR A SOURCE-DRIVEN ANTENNA WITH APPLICATION TO A FINITE CYLINDRICAL CONDUCTOR*

-Communist China-

/Following is the translation of an article by Hsü Chang-pen (1776 3864 2609), of Tsing-hua University, in Wu-li Hsüeh-pao (Journal of Physics), Vol. 12, No. 4, July 1956, pp. 281-297/

Abstract

In this article, the general boundary relations for a source-driven physical antenna are formulated in a strictly Maxwellian sense. Therefrom, by invoking the limiting condition of infinite conductivity for a lossless antenna, we arrive at a simple radiation boundary relation which brings forth the significance of the field expansions for a non-radiating source and at the same time indicates the way for a first order solution. This then characterizes the formulation of our present boundary-value problem of a "closed system" and also differentiates itself from a diffraction problem for an "open system" as far as Maxwell's field is concerned.

The formulation appears rather unfamiliar at first sight, but some reflections on the energy balance relation in macroscopic phenomena and especially on Dirac's treatment of radiation damping of election will help justify our present argument.

The first order solution for a source-driven finite cylindrical antenna obtained by using the simple radiation boundary condition gives rise to a geometrical factor which indicates the condition of resonance of the system. The necessary procedure to take into account the effect of internal impedance of a real antenna is indicated; but, thanks to the principle of superposition, this modification

* Received 20 February 1956.

does not change the essential features obtained for the case of a lossless antenna.

1. Theory of Radiation Boundary Conditions

This article -- regarding the boundary conditions of a source-driven antenna -- is rather popular. In the second and third sections, we shall use a finite cylindrical antenna to illustrate the theoretical and practical applications. The importance of this theory of boundary conditions can be explained as follows.

In the classical solution of antennae radiation problems, we assume that the electrical current possesses a sine curve distribution in the antenna and thus, we find the radiation field. This method of solution is certainly a crude approximation, since we do not consider how the sine curve current is produced or how it reacts in relation to the energy source. In this kind of solution, the matching impedance of the antenna and the energy source are taken care of individually according to the line theory. In other words, we consider the closed system of "energy source-antenna-space" as two open systems and, in solving, we do not apply Maxwell's Electro-magnetic Theory rigidly. This approximation simplifies the method of treatment and proves to be satisfactory for engineering applications.

In recent years, however, the increasingly widespread adoption of ultra-short wave and microwave has necessitated further understanding of the mutual interactions of different currents on antennae and the precise reaction of energy sources towards the current. If, as stated in the previous paragraph, we neglect the finite boundary conditions of the antenna, then it is not possible to have further understanding (assuming also that the current has a sine curve distribution in the antenna). In other words, we must take the antenna system as a boundary value problem in conjunction with an excited energy source. Hence, a forced oscillation problem of a finite conductor results. Since Maxwell and Hertz, this question has been observed in both theory and practice. Especially in more recent years, the interest for research in this area has been keener as the methods of analysis have become more precise. But it has always been neglected and not distinctly distinguished. The forced oscillation of a conductor can be divided into two kinds. The first kind is the diffraction question. The conductor, by absorbing electro-magnetic energy for the incident wave, produces forced oscillation and then radiation. The energy source of the incident waves is beyond the scope of our consideration, and is not influenced by the diffraction body. The second problem is the one that we are to discuss now. The conductor (antenna) combines direct with the energy source and is excited, producing forced oscillation, and radiates. Hence, the energy source is strongly reacted by the radiation body. The many aspects of the diffraction question have been investigated in detail by many authors, and their findings are contained in many authoratative works. But the problem that we are about to address ourselves to -- the radiation problem of the closed system of "energy source-antenna-space" -- has never received the correct theoretical development. This article may tell us the importance and significance of the two previous kinds. All in all, we can state that the diffraction problem is an open system, one of the Maxwell Electro-magnetic Field, whereas the antenna radiation discussed in this article is a closed system problem.

From the previous discussion, we know that the forced oscillation problem of a finite conductor is, in fact, a closed system of Maxwell's Electro-magnetic Field. It consists of: 1) the source; 2) radiation antenna of finite size; and, 3) infinitive, proximity space through which the electro-magnetic wave energy radiates outwardly. In such a problem, it is a simple matter to fix the finite geometrical shape and boundary of the antenna conductor. How to add energy source into the antenna is, however, not a simple problem to solve. R. King and his collaborators have made a systematic investigation of this aspect. According to the realities of physics and engineering, and the convenience of mathematical calculations, we can assume the following to be King's method: energy source is a finite potential difference V_0 added to a very small gap in the middle of the antenna. Diagram 1 represents an ex-

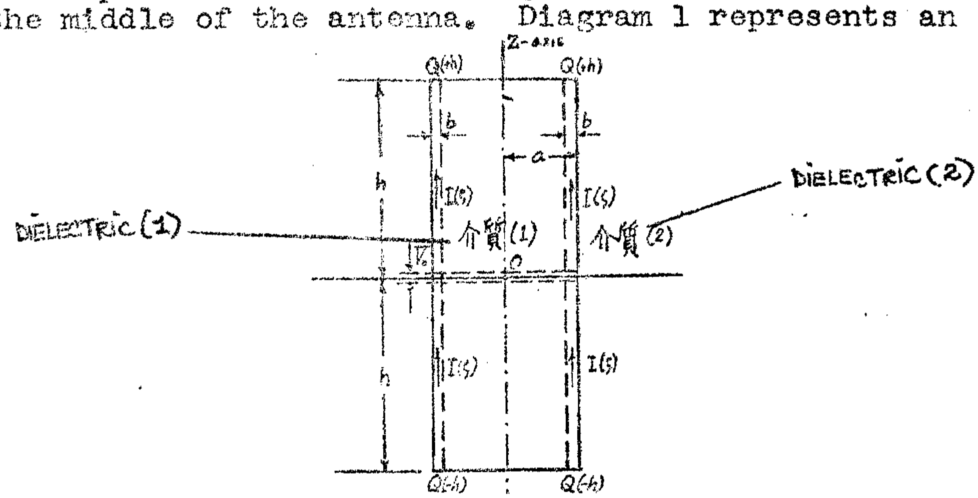

Diagram 1. Cylindrical Antenna Model.

ample of a cylindrical antenna conductor. Since the energy source gap is so small, the electrical current in the two halves of the antenna can be considered as continuous. With the above statement and understanding, we can further establish the formula of Maxwell's Electro-magnetic Field in the two media of the antenna conductor and exterior free space. Because no electro-magnetic wave energy can be transmitted to this system from outside, it must be a closed electro-magnetic field system.

For clearer understanding, this problem must adhere rigidly to the significance of the Maxwellian Field Theory. Let us look at the system of equations used by some authors. In the antenna conductor media, they use

$$\left.\begin{aligned}
&\nabla \times \dot{\mathbf{E}}_1 + \frac{\partial}{\partial t}\mathbf{B}_1 = 0\,, && \text{(a)}\\
&\nabla \times \mathbf{H}_1 - \frac{\partial}{\partial t}\mathbf{D}_1 = \mathbf{J}_1 = \sigma_1(\mathbf{E}_1' + \mathbf{E}_1)\,. && \text{(b)}\\
&\nabla \cdot \mathbf{B}_1 = 0\,, && \text{(c)}\\
&\nabla \cdot \mathbf{D}_1 = \rho_1\,. && \text{(d)}\\
&\nabla \cdot \mathbf{J}_1 + \frac{\partial}{\partial t}\rho_1 = 0\,. && \text{(e)}
\end{aligned}\right\} \quad (1)$$

E_1, D_1, H_1, and B_1, are the electro-magnetic field produced by induction current J_1 and charge ρ_1, on the conductor. E_1 is assumed to be the only field produced by the energy source. From the strictly Maxwellian Field standpoint of the incompleteness and the inappropriateness of the equational system (1) are obvious. Regardless of how the high-frequency energy source is added to the antenna conductor, it produces not only a field E'_1, but also, simultaneously, displacement current $\frac{\partial}{\partial t}D'_1$ and equivalent magnetic fields H_1 and B_1. According to the principle of superposition, the complete Maxwellian Electro-magnetic Field equation system in the conductor dielectric must be:

$$\left.\begin{aligned}
&\nabla \times \mathbf{E}_1' + \frac{\partial}{\partial t}\mathbf{B}_1' = 0\,, \quad \nabla \times \mathbf{E}_1 + \frac{\partial}{\partial t}\mathbf{B}_1 = 0\,, && \text{(a)}\\
&\nabla \times [\mathbf{H}_1' + \mathbf{H}_1] - \frac{\partial}{\partial t}[\mathbf{D}_1' + \mathbf{D}_1] = \mathbf{J}_1 = \sigma_1[\mathbf{E}_1' + \mathbf{E}]\,. && \text{(b)}\\
&\nabla \cdot \mathbf{B}_1' = 0\,, \quad \nabla \cdot \mathbf{B}_1 = 0\,, && \text{(c)}\\
&\nabla \cdot [\mathbf{D}_1' + \mathbf{D}_1] = \rho_1\,, && \text{(d)}\\
&\nabla \cdot \mathbf{J}_1 + \frac{\partial}{\partial t}\rho_1 = 0\,; && \text{(e)}
\end{aligned}\right\} \quad (2)$$

where fields with prime are produced by the energy source, and fields without prime represent the reaction in the conductor dielectric. The linear interaction between energy source and reaction will be determined entirely from the formulae (2b) and (2d), and produce, in the conductor, the combined current J_1 and charge ρ_1.

In the external free-space dielectric (2) -- since its conductivity equals zero -- neither the conduction current nor the charge can exist. Therefore, interaction formulae (2b) and (2d) can be separated, arriving at the following combined system of equations of two independents -- energy source field and induction field:

$$\left.\begin{array}{lll} \nabla \times \mathbf{E}_2' + \frac{\partial}{\partial t}\mathbf{B}_2' = 0, & \nabla \times \mathbf{E}_2 + \frac{\partial}{\partial t}\mathbf{B}_2 = 0, & \text{(a)} \\ \nabla \times \mathbf{H}_2' - \frac{\partial}{\partial t}\mathbf{D}_2' = 0, & \nabla \times \mathbf{H}_2 - \frac{\partial}{\partial t}\mathbf{D}_2 = 0, & \text{(b)} \\ \nabla \cdot \mathbf{B}_2' = 0, & \nabla \cdot \mathbf{B}_2 = 0, & \text{(c)} \\ \nabla \cdot \mathbf{D}_2' = 0, & \nabla \cdot \mathbf{D}_2 = 0. & \text{(d)} \end{array}\right\} \quad (3)$$

From the Maxwellian field equation system of induction fields in the two dielectrics and energy source, the usual boundary relation tells us that on the boundary surface of the conductor, the tangential component of the electric field and magnetic field of two dielectrics must be continuous:

$$\left.\begin{array}{l} [\mathbf{H}_1'+\mathbf{H}_1]_t = [\mathbf{H}_2'+\mathbf{H}_2]_t, \\ [\mathbf{E}_1'+\mathbf{E}_1]_t = [\mathbf{E}_2'+\mathbf{E}_2]_t. \end{array}\right\} \quad (4)$$

Theoretically speaking, if the expansion form $E'_{1,2}$ and $H'_{1,2}$ of the source field of two dielectrics is known, we can definitely find out the induction field $E_{1,2}$ and $H_{1,2}$, and the induction current and charges on the conductor from formula (4). However, the importance and meaning of the usual boundary relation in (4) is not in the mathematical technique of expanding the source field, but in the basic concept of interchanging action between the source field and the induction field. Using the ideal limiting conditions of infinitely large conductivity of the conductor, we can easily understand this concept. When the conductivity (of a conductor) is infinitely large, the electro-magnetic field in the conductor is equal to zero, and formula (4) is simplified according to the following form:

$$\left.\begin{aligned} 0 &= [\mathbf{H}_2' + \mathbf{H}_2]_r , \\ 0 &= [\mathbf{E}_2' + \mathbf{E}_2]_r . \end{aligned}\right\} \tag{5}$$

Since energy source and induction field are the continuous functions of space and time, relations of formula (5) are accurate not merely in boundary surfaces of two dielectrics, but also at every point in the external free-space. Thus, because the components of electro-magnetic field have a singular nature, we can arrive at the following "no loss" general relations for an antenna:

$$\mathbf{H}_2' + \mathbf{H}_2 = 0 , \qquad \mathbf{E}_2' + \mathbf{E}_2 = 0 . \tag{6}$$

Hence, source field and induction field simply cancel out each other in outer free-space. That is not to say, however, that an excited "no loss" antenna does not discharge radiation. As a matter of fact, according to naturally logical results obtained from the Maxwell Field Theory, it should be regarded as the equilibrium energy relationship between energy source and response. Similarly, in formula (4), in respect to an antenna with limited electrical conductivity, the energy source, besides providing radiation energy, also has a field to overcome the internal impedance of an antenna. Since the Maxwellian equation implies the principle of energy equilibrium we can, therefore -- to boundary relationships obtained by closed systems of an energy-included source -- imply the same principle. From this, it can be seen that our derivation and explanation above have not established any new concept outside the Maxwellian system. Energy source field E'_2 and H'_2 are merely a mathematical guide field and induction field E_2 and H_2 must react according to its model. This is the same as the Newtonian principle of "an equal, and opposite, reaction for every action". This is the characteristic of the logical derivation according to the Maxwellian "closed" theory, and tells us clearly that the energy source provides merely radiation energy, but does not itself produce radiation directly. The actual radiation source is the induction current having form on the antenna. At the same time, the above derivation distinguishes clearly the problem we are discussing at present and general diffraction problems. In the diffraction problem, the conductor receives electro-magnetic wave energy from an external energy source; hence, it is an open system -- looking at it from the point of view of the Maxwellian Field Theory. In the problem we are now discussing, however, conductor absorbs energy directly from the energy source, and reacts to the energy source according to the geometric forms and physical properties of that conductor, while the energy source itself does not produce radiation. Hence, regarding this from the standpoint of the Maxwellian Field Theory, "energy source-antenna-free space" does constitute a closed system.

Therefore, formula (6) is the general radiation boundary relationship of a non-radiation energy source excitation for a "no loss" antenna. Because the energy source does not produce an electro-magnetic field in an ideal conductor antenna, the energy source field E'_2 and H'_2 of outer free-space can be computed very easily from the determined form of energy source. In the following two sections, we will use this radiation boundary relationship, solve the problem of a finite cylindrical antenna with an internal energy source potential difference V_o. As to the effects of the internal impedance inside the antenna, we can fill it up with suitable radial distribution of the energy source voltage.

Having understood the concept of this derivation, and the character of meaning of the boundary relationships of an ideal "no loss" antenna, we can further find the complete meaning of formula (4) for the application of finite electrical conductivity antennae. Since the meaning of the two equations of relationship (4) are the same, it suffices if we only discuss the second equation concerning the field. From formulae (5) and (6), it can be seen that the radiation components of E'_2 and E_2 exactly cancel out each other. Hence, what remains on the right side of formula (4) is only the "residual" of "local" field. Suppose we let

$$\left.\begin{aligned} E'_2 &= E'_{2(rad)} + E'_{2(local)}, \\ E_2 &= E_{2(rad)} + E_{2(local)}, \end{aligned}\right\} \tag{7}$$

And, from formulae (5) and (6), the radiation fields are naturally cancelled out:

$$E'_{2(rad)} + E_{2(rad)} = 0 \tag{8}$$

Then, the second formula in relationship (4) can be much simplified by the following form:

$$[E'_1 + E_1]_r = [E'_{2(local)} + E_{2(local)}]_r \tag{9}$$

From the two simultaneous equations in formulae (8) and (9), if the energy source field is known theoretically, then we can completely determine the radiation component and the local component of the induction field E_2. But, this method of finding a solution has two serious drawbacks: first, even if the energy source voltage has very simple antennae specifications in free-space, the electromagnetic field produced by the energy source is still very hard to find; second, to solve the simultaneous differential equations of formulae (8) and (9), though not entirely immpossible, it is very complicated. Because of these difficulties, we have to rely upon the simple radiation boundary relationships of formulae (5) and (6), in order to solve the radiation problems in outer free-space in a "no loss" antenna. Afterwards, we again use the disturbance method to calculate the effect of its internal impedance in a "no loss" antenna.

From what has been described above, the deduced physical meaning of our accurate derivation -- ripe with implications -- is also very important. Concerning the fundamental concept of the production and the action of energy source field and induction field, with the exception of what was discussed above, now this "closed derivation" shows the meaning of intereffects between energy source and induction. We can limit the form of energy source, but before knowing the accurate reaction function, we cannot limit the expanded function of the field. They must be internally compatible in the Maxwellian system, and conform to the principle of linear superposition included in formulae (2) and (3). Naturally, this is the characteristic of closed deduction for any physical phenomenon. Einstein's deductive Theory of General Attraction is an example. In our present problem, as long as the actual space dielectric has a little loss, then the energy field and reaction field interact and interhinge through the whole time-space system. Hence, it is very difficult -- almost impossible -- under the general conditions, to find the accurate equilibrium state of energy field and reaction field in the Maxwellian system. Fortunately, under ideal limiting conditions of "no loss" antenna and free space, we get very simple formulae of radiation boundary conditions, and their solutions are obvious.

2. Symbols and Terms in Energy Source and Antennae Systems

For reference, see Diagram 1 (page 3). Symbols and terms used in this article can be described as follows:

2h..................Total length of cylindrical antenna;
a...................radius of antenna;
b...................thickness of very thin cylindrical surface layer -- high-frequency cur-

rent flows through this thin layer, $b \ll a$;

V_0 .... potential difference of energy source (or thin-plate voltage source) of circular symmetry;

$(\pm 0, \rho_0, \varphi_0)$ .... in layer b, co-ordinates of thin-plate voltage source, $\rho_0 \doteqdot a$;

$(\zeta, \rho_0, \varphi_0)$ .... co-ordinate of circular symmetry induction current in layer b, $\rho_0 \doteqdot a$;

(z, r, θ) .... co-ordinate of observation point P;

$(\pm h, \rho_0, \varphi_0)$ .... co-ordinate of points of the cylindrical-base charges $Q(\pm h)$, $\rho_0 \doteqdot a$;

$I(\zeta)$ .... average induction current density in layer b -- its equivalent volume charge density is q;

$Q(\pm h)$ .... average base charge density;

$e^{-j\omega t}$ .... exponential time co-efficient;

$k_1^2 = \omega^2 \mu_1 \varepsilon_1 + j \omega \mu_1 \sigma_1$, .. propagational constant of dielectric (1) of the circular conductor;

$k_2^2 = \omega^2 \mu_2 \varepsilon_2 + j \omega \mu_2 \sigma_2$, .. propagational constant of dielectric (2) of outside free-space;

$$R_0 = [r^2 + \rho_0^2 - 2 r \rho_0 \cos (\varphi_0 - \theta) + (z-0)^2]^{\frac{1}{2}},$$

distance from observation point P to every electric source;

$$R_1 = [r^2 + \rho_0^2 - 2 r \rho_0 \cos (\varphi_0 - \theta) + (z - \zeta)^2]^{\frac{1}{2}},$$

distance from observation point P to every induction current;

(10) $$R_2 = [r^2 + \rho_0^2 - 2 r \rho_0 \cos (\varphi_0 - \theta) + (z - h)^2]^{\frac{1}{2}},$$

distance from observation point P to base charges Q(+h);

$$R_3 = [r^2 + \rho_0^2 - 2 r \rho_0 \cos (\varphi_0 - \theta) + (z + h)^2]^{\frac{1}{2}},$$

distance from observation point P to base charge Q(-h);

(11) $$d\Pi_0 = \hat{z} e^{-j\omega t} \frac{V_0 b \rho_0 d\varphi_0}{4 \pi R_0} e^{jk_2 R_0},$$

expansion form of Hertz function for every component $(\pm 0, \rho_0, \varphi_0)$ of the voltage source (about its derivation, see Appendix);

(12) $$d^2\Pi = \hat{z} e^{-j\omega t} \frac{I(\zeta)\, d\zeta\, \rho_0\, d\varphi_0}{-j 4 \pi \omega \varepsilon_2 R_1} e^{jk_2 R_1},$$

expansion form for all points of the Hertz function of the induction current $I(\zeta) d\zeta$.

$$\left.\begin{aligned} \frac{\partial}{\partial t} Q(+h) &= -j\omega Q(+h) = I(+h), \\ \frac{\partial}{\partial t} Q(-h) &= -j\omega Q(-h) = -I(-h) = -I(+h), \end{aligned}\right\} \qquad (13)$$

the continuity equation of base charges $Q(\pm h)$;

$$d\phi = e^{-j\omega t} \frac{Q(+h)\, b\, \rho_0\, d\varphi_0}{4\pi \varepsilon_2 R_2} e^{jk_2 R_2} + e^{-j\omega t} \frac{Q(-h)\, b\, \rho_0\, d\varphi_0}{4\pi \varepsilon_2 R_3} e^{jk_2 R_3} = \qquad (14)$$

$$= e^{-j\omega t} \frac{I(+h)\, b\, \rho_0\, d\varphi_0}{-j4\pi\omega \varepsilon_2 R_2} e^{jk_2 R_2} + e^{-j\omega t} \frac{I(+h)\, b\, \rho_0\, d\varphi_0}{j4\pi\omega \varepsilon_2 R_3} e^{jk_2 R_3},$$

the two base charges $Q(\pm h)$ equivalent to the scalar potential function for every point.

In this article, M.K.S. units are employed to calculate all formulae and field quantities.

3. Theory of Energy Source Excitation of a Finite Cylindrical Antenna.

Diagram 1 shows a finite cylindrical antenna inside of which there is a thin plate of voltage source. Under very high frequency, it can be proved that the current along the cylindrical axis is limited in a very thin surface layer. Since our present interest is the investigation of high-frequency energy source excited antennae, we can assume the thickness of that current surface layer to be b, where b is much smaller than the radius of the cylinder, a. Therefore, the distance between the thin current layer and the cylinder axis, can be said to be equal to radius a, as indicated in the formulae in the second section. Although the radial current in the antenna is not entirely equal to zero, it can be completely disregarded in practical calculation of radiation fields when compared with the main axial current in layer b. Since axial current is not absolutely equal to zero at the two bases of the cylinder, we can naturally assume that surface charges exist on them. They also fulfill the equation of continuity formula (13) above. The simple model above accurately represents the actual situation of a finite cylindrical antenna under high-frequency energy excitation. At the same time, it has singularness in the sense of the Maxwellian system.

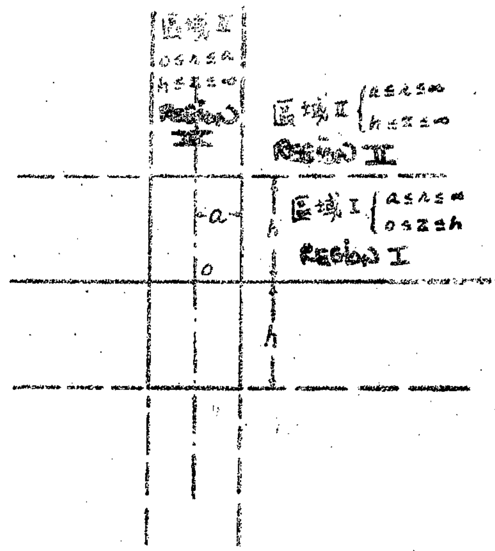

Diagram 2. Region of Integration.

Now we can use the radiation boundary relationship (6), and the simple model indicated in Diagram 1, in order to calculate the field components along the z axis.

$$[E_z'+E_z]_z = \frac{\partial^2}{\partial z^2}[\Pi_0+\Pi] + k_2^2[\Pi_0+\Pi] - \frac{\partial}{\partial z}\phi = 0. \qquad (15)$$

where Π_o is the total Hertz function of the energy source Π is the Hertz function produced by the density q of the induction current and volume charges. ϕ is the total scalar electrical potential produced by the charges on the two base surfaces. Since the radial, or r-component, of the field can be accurately calculated from its z-component using Maxwell's field equation -- it will suffice if we calculate the z-component of formula (15).

The differential forms of the three functions Π_o, Π, and ϕ have been expressed in formulae (11), (12), and (14) -- in the second section. Due to the fact that the energy source, induction current and charges on base surfaces of the two half-sections of the antenna are anti-symmetrical, we need only compute the field along the positive z axis. Diagram 2 shows three different Regions. Within each region, different potential functions must be accurately expanded. The following is their derivation:

We expand the Hertz function Π_o of the energy source in the following manner:

$$\frac{e^{ik_2R_0}}{R_0} = \frac{i}{2}\int_{-\infty}^{+\infty} H_0^{(1)}\left(\sqrt{k_2^2-\lambda^2}\,\sqrt{r^2+\rho_0^2-2r\rho_0\cos(\varphi_0-\theta)+(z-0)^2}\right) e^{i\lambda z}\,d\lambda =$$

$$= \begin{cases} \sum_n \frac{j}{2} \int_{-\infty}^{+\infty} J_n(\rho_0 \sqrt{k_2^2-\lambda^2})\, H_n^{(1)} (r\sqrt{k_2^2-\lambda^2})\, e^{jn(\varphi_0-\theta)}\, e^{j\lambda z}\, d\lambda, & r > \rho_0 \\ \sum_n \frac{j}{2} \int_{-\infty}^{+\infty} J_n(r \sqrt{k_2^2-\lambda^2})\, H_n^{(1)} (\rho_0\sqrt{k_2^2-\lambda^2})\, e^{jn(\varphi_0-\theta)}\, e^{j\lambda z}\, d\lambda, & r < \rho_0. \end{cases} \quad (16)$$

Substituting formula (16) into formula (11), and integrating with respect to φ_0 from 0 to 2π, we get the Hertz function of total energy source:

$$\Pi_0 = \begin{cases} e^{-j\omega t}\left(\frac{jV_0}{4}\right)\int_{-\infty}^{+\infty} b\,\rho_0\, J_0(\rho_0 \sqrt{k_2^2-\lambda^2})\, H_0^{(1)} (r\sqrt{k_2^2-\lambda^2})\, e^{j\lambda z}\, d\lambda, & r > \rho_0, \\ e^{-j\omega t}\left(\frac{jV_0}{4}\right)\int_{-\infty}^{+\infty} b\,\rho_0\, J_0(r \sqrt{k_2^2-\lambda^2})\, H_0^{(1)} (\rho_0\sqrt{k_2^2-\lambda^2})\, e^{j\lambda z}\, d\lambda. & r < \rho_0. \end{cases} \quad (17)$$

We expand the Hertz function II of induction current $I(\zeta)$

$$\frac{e^{jk_2R_1}}{R_1} = \frac{j}{2}\int_{-\infty}^{+\infty} H_0^{(1)} (\sqrt{k_2^2-\lambda^2}\,\sqrt{r^2+\rho_0^2-2r\rho_0\cos(\varphi_0-\theta)+(z-\zeta)^2})\, e^{j\lambda|z-\zeta|}\, d\lambda =$$

$$\begin{cases} \sum_n \frac{j}{2} \int_{-\infty}^{+\infty} J_n(\rho_0 \sqrt{k_2^2-\lambda^2})\, H_n^{(1)} (r\sqrt{k_2^2-\lambda^2})\, e^{jn(\varphi_0-\theta)}\, e^{j\lambda|z-\zeta|}\, d\lambda, & r > \rho_0, \\ \sum_n \frac{j}{2} \int_{-\infty}^{+\infty} J_n(r \sqrt{k_2^2-\lambda^2})\, H_n^{(1)} (\rho_0\sqrt{k_2^2-\lambda^2})\, e^{jn(\varphi_0-\theta)}\, e^{j\lambda|z-\zeta|}\, d\lambda, & r < \rho_0. \end{cases} \quad (18)$$

Substituting formula (18) into formula (12), and integrating with respect to φ_0, we obtain

$$d\Pi = \begin{cases} e^{-j\omega t}\,\frac{I(\zeta)d\zeta}{-4\omega\varepsilon_2}\int_{-\infty}^{+\infty} b\,\rho_0\, J_0(\rho_0 \sqrt{k_2^2-\lambda^2})\, H_0^{(1)} (r\sqrt{k_2^2-\lambda^2})\, e^{j\lambda|z-\zeta|}\, d\lambda, & r > \rho_0 \\ e^{-j\omega t}\,\frac{I(\zeta)d\zeta}{-4\omega\varepsilon_2}\int_{-\infty}^{+\infty} b\,\rho_0\, J_0(r \sqrt{k_2^2-\lambda^2})\, H_0^{(1)} (\rho_0\sqrt{k_2^2-\lambda^2})\, e^{j\lambda|z-\zeta|}\, d\lambda, & r < \rho_0. \end{cases} \quad (19)$$

In our present problem, we need only integrate formula (19) in Region II with respect to ζ, and we have

$$\Pi = e^{j\omega t}\left(\frac{1}{-2\omega\varepsilon_2}\right)\int_{-\infty}^{+\infty} b\,\rho_0\, J_0(\rho_0\sqrt{k_2^2-\lambda^2}) \times \quad (20)$$
$$\times H_0^{(1)} (r\sqrt{k_2^2-\lambda^2})\, d\lambda\, e^{j\lambda z}\left[\int_0^h I(\zeta)\cos\lambda\zeta\, d\zeta\right]$$

We expand the scalar potential function ϕ, produced by the charges on the base surface

$$\frac{e^{jk_2R_{2,3}}}{R_{2,3}} = \begin{cases} \sum_n \frac{j}{2} \int_{-\infty}^{+\infty} J_n(\rho_0 \sqrt{k_2^2-\lambda^2})\, H_n^{(1)} (r\sqrt{k_2^2-\lambda^2})\, e^{jn(\varphi_0-\theta)}\, e^{j\lambda|z\mp h|}\, d\lambda, & r > \rho_0 \\ \sum_n \frac{j}{2} \int_{-\infty}^{+\infty} J_n(r \sqrt{k_2^2-\lambda^2})\, H_n^{(1)} (\rho_0\sqrt{k_2^2-\lambda^2})\, e^{jn(\varphi_0-\theta)}\, e^{j\lambda|z\mp h|}\, d\lambda, & r < \rho_0. \end{cases} \quad (21)$$

Substituting formula (21) in formula (14), and integrating with respect to φ_0, we get the function ϕ in Region II

$$\phi = e^{-j\omega t}\left(\frac{1}{-2\omega\varepsilon_2}\right)\int_{-\infty}^{+\infty} b\,\rho_0\,J_0(\rho_0\sqrt{k_2^2-\lambda^2}) \times \times H_0^{(1)}(r\sqrt{k_2^2-\lambda^2})\,e^{j\lambda z}\,[-I(h)\,j\,\sin\lambda h]\,d\lambda. \tag{22}$$

Substituting formulae (17), (20), and (22) into radiation boundary relationship (15), we get the following identical equation in Region II

$$e^{-j\omega t}\frac{1}{2}\int_{-\infty}^{+\infty} b\,\rho_0\,J_0(\rho_0\sqrt{k_2^2-\lambda^2})\,H_0^{(1)}(r\sqrt{k_2^2-\lambda^2})\,d\lambda\,e^{j\lambda z} \times \times\left\{(k_2^2-\lambda^2)\left[\frac{V_0}{2}-\frac{1}{j\omega\varepsilon_2}\int_0^h I(\zeta)\cos\lambda\zeta\,d\zeta\right]+\frac{\lambda\sin\lambda h}{j\omega\varepsilon_2}I(h)\right\}=0. \tag{23}$$

In Region I, we can also obtain the same identical equation, but to integrate with respect to ζ is a very complicated process. We will not discuss it further here. In equation (23), as long as the functional constant in the bracket is zero, then formula (23) in the radiation field, or on the boundary, will in any case be completely fulfilled. Hence, we have the following relationship

$$\int_0^h I(\zeta)\cos\lambda\zeta\,d\zeta = \frac{j\omega\varepsilon_2 V_0}{2}+\frac{\lambda\sin\lambda h}{k_2^2-\lambda^2}I(h) \tag{24}$$

in which we can compute the function $I(\zeta)$ of current distribution for a "no loss" antenna in a simple model as ζ shown in Diagram 1. From the simple model of this antenna in Diagram 1, we know that $I(\zeta)$ is the even function of λ. It must also be the even function of λ, for if $I(\zeta)$ is the odd function of λ, then the integration of the constant induction Hertz function II with respect to will make II equal to zero. From the above, we can assume the solution of $I(\zeta)$ in formula (24) to be

$$I(\zeta) = \frac{j\omega\varepsilon_2 V_0}{h}\,G(\lambda)\cos\lambda\zeta\,; \tag{25}$$

where $G(\lambda)$ must be an even function of λ, and is dimensionless. Substituting formula (25) into the two sides of formula (24), while integrating with respect to ζ , we get the solution of $G(\lambda)$:

$$G(\lambda) = \frac{2\lambda h}{2\lambda h+\sin 2\lambda h}\left[1-\frac{\lambda\sin\lambda h}{k_2^2-\lambda^2}\cdot\frac{4\lambda}{2\lambda h+\sin 2\lambda h}\cos\lambda h\right]. \tag{26}$$

Hence, $G(\lambda)$ is actually the even function of ζ and λ, simultaneously. Here we should notice the following:

formula (25) is still the operational function of I(ζ). Its operational parameter is λ. When we substitute I(ζ) of formula (25) into formulae (20) and (22), and after integrating with respect to λ, we get the algebraic solution of induction potential function. Substituting these II and ϕ algebraic solutions,

$$\left.\begin{aligned} &\mathbf{E}_2 = \nabla\nabla\cdot\mathbf{\Pi} - \mu_2\varepsilon_2\frac{\partial^2}{\partial t^2}\mathbf{\Pi} - \nabla\phi \\ &\frac{\partial}{\partial t}\mathbf{B}_2 = \mu_2\varepsilon_2\nabla\times\left(\frac{\partial^2}{\partial t^2}\mathbf{\Pi}\right) \quad 或 \quad \mathbf{H}_2 = j\omega\varepsilon_2\nabla\times\mathbf{\Pi}, \end{aligned}\right\} \tag{27}$$

we get the algebraic solution of field E_2 and H_2 of the space dielectric. Finally, from the following simple formula,

$$I(z) = \frac{1}{b}\,[H_{2\theta}]_{r=\rho_0} = \frac{-j\omega\varepsilon_2}{b}\left[\frac{\partial}{\partial r}\mathbf{\Pi}\right]_{r=\rho_0}, \tag{28}$$

we get the algebraic solution of current distribution on the antenna. The above procedure for finding a solution is the special characteristic of finding the integral over a complex number surface (λ-surface here) of any boundary value problem. In our present problem, the harmonics, or standing waves, caused by the finite boundaries of an antenna conductor, will show its form only after the operational form has changed from -integral to algebraic form.

From formula (28), after arriving at the current distributional function, we can give the following definitive relationship of the induction point admittance or impedance of a "no loss" antenna:

$$Y_0 = \frac{1}{Z_0} = \frac{2\pi\rho_0 b}{V_0}\,I(0) = \frac{-j\,2\pi\rho_0\omega\varepsilon_2}{V_0}\left[\frac{\partial}{\partial r}\Pi\right]_{z=0,\ r=\rho_0}: \tag{29}$$

where Y and Z, respectively, are the induction admittance and impedance of the energy source in the center of the antenna.

Calculation of II function in Region I. Now we can substitute I(ζ) in formula (25) into formula (19), and integrate in Region I with respect to ζ, since I($-\zeta$) I(ζ). We have only to calculate the range of $0<z<h$, equal to the upper-half region of the antenna; and, in this z range, the operational parameter must have a positive imaginery number component.

$$\Pi = e^{-j\omega t}\left(\frac{jV_0}{-4h}\right)\int_{-\infty}^{+\infty} b\,\rho_0\,J_0(\rho_0\sqrt{k_2^2-\lambda^2})\,H_0^{(1)}(r\sqrt{k_2^2-\lambda^2})\,G(\lambda)\,d\lambda\left[\int_{-h}^{+h} e^{j\lambda|z-\zeta|}\cos\lambda\zeta\,d\zeta\right] =$$

$$= \cdots \int \cdots d\lambda \left[2\, e^{j\lambda z} \int_0^z (\cos \lambda\zeta)^2\, d\zeta + 2 \cos \lambda z \int_z^h e^{j\lambda\zeta} \cos \lambda\zeta\, d\zeta \right] =$$

$$= \cdots \int \cdots d\lambda \left[2 \cos \lambda z \int_0^h (\cos \lambda\zeta)^2\, d\zeta \right] = \cdots \int \cdots d\lambda \left[\frac{2 \cos \lambda z}{4\lambda} (2\lambda h + \sin 2\lambda h) \right]. \tag{30}$$

The last two lines can be obtained from the properties of even and odd functions by integrating with respect to λ. The factor in formula (30),

$$2e^{-j\omega t} \cos \lambda z = e^{j(\lambda z - \omega t)} + e^{-j(\lambda z + \omega t)},$$

clearly points to the following: this Hertz induced potential function or current is a standing wave; that is to say, the multi-reflection waves of different Eigen values to λ. This is again just what we would expect to be the result, as far as the effect of the two flat base surfaces of a "no loss" antenna is concerned. Since the integration quantity in formula (30), with respect to λ, has an even functional property, the λ-surface integral calculation can be written into the following form:

$$\Pi = e^{-j\omega t} \left(\frac{j V_0}{-4} \right) \int_{-\infty}^{+\infty} b\, \rho_0\, J_0(\rho_0 \sqrt{k_2^2 - \lambda^2})\, H_0^{(1)} (r \sqrt{k_2^2 - \lambda^2}) \times$$
$$\times\, e^{j\lambda z} \left[1 - \frac{2\lambda^2 \sin 2\lambda h}{(k_2^2 - \lambda^2)(2\lambda h + \sin 2\lambda h)} \right]^{-1} d\lambda. \tag{31}$$

As to the scalar potential function ϕ, caused by the charges on the two bases, it has the following form:

$$\phi = e^{-j\omega t} \left(\frac{j V_0}{-4h} \right) \int_{-\infty}^{+\infty} b\, \rho_0\, J_0(\rho_0 \sqrt{k_2^2 - \lambda^2})\, H_0^{(1)} (r \sqrt{k_2^2 - \lambda^2}) \times$$
$$\times\, e^{j\lambda z} \left[\frac{2\lambda h + \sin 2\lambda h}{2\lambda h \sin 2\lambda h} - \frac{\lambda h}{h^2 (k_2^2 - \lambda^2)} \right] d\lambda. \tag{32}$$

The integral quantity in formula (31) shows that we can add an infinitely large half-circumference on the upper-half of the λ-surface, in order to make a closed-curve integral, without affecting its result. As shown in Diagram 3 (see next page):

$$\lambda = + k_2 = + (\omega^2 \epsilon_2 \mu_2 + j\omega \mu_2 \epsilon_2)^{1/2} \Big|_{\epsilon_2 \to 0}$$

point is a branch point on the closed integral curve Γ, but $\lambda = -k_2$ point should be outside of the closed curve Γ. The integral quantity of formula (31), except outside the branch point $\lambda = -k_2$, still has the following transcendental equation

$$1-\frac{2\lambda^2\sin 2\lambda h}{(k_2^2-\lambda^2)\,(2\lambda h+\sin 2\lambda h)}=0 \quad 或 \quad \sin 2\lambda h=\frac{2\lambda h(k_2^2-\lambda^2)}{3\lambda^2-k_2^2}, \qquad (33)$$

the roots of which have numerous simple poles. We can very easily prove that $\lambda=0$ is not a root of formula (33), and that all its roots must have the positive and negative component of complex numbers. Let λ'_ms represent the different complex roots of transcendental equation (33), and let Br represent the suitable branchcut from $\lambda=+k_2$ to infinity. The integral result of formula (31) can be written in the following manner:

$$II=\cdots\cdots\oint_{\Gamma}\cdots\cdots d\lambda=\cdots\cdots\int_{Br.}\cdots\cdots d\lambda+2\pi j\sum_{m} R_m(\lambda_m) \qquad (34)$$

The R'_ms residues at different λ'_ms poles all have a negative expotential damping factor. That is why these similar residue waves are really the localized waves concentrating at the energy source point ($z=\pm 0$). At points on the antenna a bit too far away from the energy source point (or surface), these residue waves become very weak as a result of damping. Hence, at different points where $z\neq\pm 0$ on the antenna, II -- the main component of the integral form -- is integrated along the branchcut, and these resultant residue waves, R'_ms, can be entirely disregarded in the calculation. But the accurate calculation of these

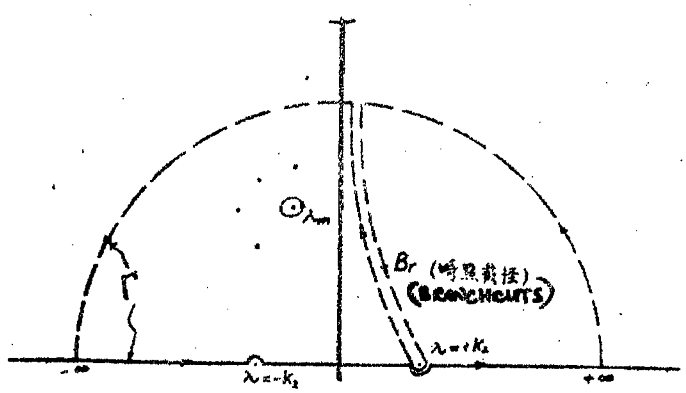

Diagram 3. λ-surface Integral Curve.

branchcut integrals is still very complicated. So, in discussing the following two conditions,

(next page)

$$\left.\begin{array}{lll} \text{a)} & \text{Non-harmonic conditions} & : 2k_2 h \neq n, \text{ or } h \neq \frac{n\lambda_0}{4} \\ \text{b)} & \text{Harmonic conditions} & : 2k_2 h = n, \text{ or } h = \frac{n\lambda_0}{4} \end{array}\right\} n = 1, 2, 3 \cdots, \qquad (35)$$

We will use an appropriate approximation method. The λ_0 in formula (35), represents the wave length of energy source frequency in free space.

a) Non-harmonic conditions. From formulae (31) and (34), and from Diagram 3, we note that when we ascend along the branchcut Br, larger and larger negative expotential damping factors make the resulting main component of the integral approach k_2 from λ. Then, in the vicinity of $\lambda \doteqdot k_2$, we can remove very slowly the factors in the variable values in the integral sign. Thus, we obtain

$$\Pi = e^{-i\omega t}\left(\frac{jV_0 \rho_0 b}{8k_2^2}\right)\frac{2k_2 h + \sin 2k_2 h}{\sin 2k_2 h}\int_{Br}(k_2^2-\lambda^2)\, H_0^{(1)}\left(r\sqrt{k_2^2-\lambda^2}\right) e^{j\lambda z}\, d\lambda \qquad (36)$$

where the term $J_0\left(\rho_0\sqrt{k_2^2-\lambda^2}\right)$ in formula (31), in the vicinity of $\lambda \doteqdot k_2$, can be treated as constants $J_0 = 1$. The formula thus obtained, has already no pole inside the integral sign. Hence, we can place the branchcut integral Br back into the actual real number axis position, getting the following result:

$$\begin{aligned} \Pi &= e^{-j\omega t}\left(\frac{jV_0 \rho_0 b}{8k_2^2}\right)\frac{2k_2 h + \sin 2k_2 h}{\sin 2k_2 h}\int_{-\infty}^{+\infty}(k_2^2-\lambda^2)\, H_0^{(1)}\left(r\sqrt{k_2^2-\lambda^2}\right) e^{j\lambda z}\, d\lambda = \\ &= e^{-j\omega t}\left(\frac{V_0 \rho_0 b}{2}\right)\left(\frac{2k_2 h + \sin 2k_2 h}{2k_2^2 \sin 2k_2 h}\right)\left(k_2^2 + \frac{\partial^2}{\partial z^2}\right)\frac{j}{2}\int_{-\infty}^{+\infty} H_0^{(1)}\left(r\sqrt{k_2^2-\lambda^2}\right) e^{j\lambda z}\, d\lambda = \\ &= e^{-j\omega t}\left(\frac{V_0 \rho_0 b}{2}\right)\left(\frac{2k_2 h + \sin 2k_2 h}{2k_2^2 \sin 2k_2 h}\right)\left(k_2^2 + \frac{\partial^2}{\partial z^2}\right)\left[\frac{e^{jk_2\sqrt{r^2+z^2}}}{\sqrt{r^2+z^2}}\right]. \end{aligned} \qquad (37)$$

where the last line in the formula is obtained from the spherical waves from Fouriers' Transformation, at $r>0$, and any value of z, have equal convergent properties. Hence, it is differentiable with respect to z.

b) Harmonic conditions. Suppose we do not need a more accurate calculation of formula (31), then under the harmonic conditions its solution, naturally, is also formula (37). But, it is in harmonic state $2k_2 h = n\pi$; hence, the following term of factor becomes infinitely large:

$$\frac{2k_2 h + \sin 2k_2 h}{2k_2^2 \sin 2k_2 h} = 2h^2\left(\frac{n\pi + \sin n\pi}{n^2\pi^2 \sin n\pi}\right) = \infty. \qquad (37a)$$

This simply shows the harmonic phenomenon of the energy source frequency for this "no loss" antenna system in a vacuum. It can be seen that in this theory, in the process of finding a solution using approximations this important conclusion has not been missed. It is the harmonic reaction of energy source of a closed system with which the antenna and space are connected. This well-known phenomenon, and its engineering applications, have anyhow its simple mathematical proof. There must exist "no loss" conditions in the real universe space we are in, and the discharged electro-magnetic wave energy must also be absorbed entirely or converted into another energy. Hence, k_2^2 must have a positive imaginery number component:

$$k_2^2 = \omega^2 \mu_2 \varepsilon_2 + j \omega \mu_2 \sigma_2 ;$$

where $\sigma_2 = 0$, but, however, is very small. So, the harmonic factor in the above formula (37a), though very large, will not become infinity. In our simple theory, the natural appearance of the harmonic phenomenon of this closed system -- "energy-antenna-space" -- is very fortunate.

Calculation of ϕ function caused by the charges on the two bases. Similarly, in the integral of the scalar potential function ϕ in formula (32), we arrive at the cut radius integral which goes around the branch point $\lambda = +k_2$, and the poles of numerous λ'_ms complex numbers of formula (33). The residue waves obtained by the integral surrounding these λ'_ms poles actually disappear little by little as they go away from the energy source ($z = \pm 0$) on the antenna. The main component of the integral is obtained from the branchcut $\lambda = +k_2$. Using a method similar to the previous one in calculating the II function, we can obtain the following:

$$\phi = e^{-i\omega t}\left(\frac{-j V_0 \rho_0 b}{2 k_2}\right)\left(k_2^2 + \frac{\partial^2}{\partial z^2}\right)\left[\frac{e^{jk_2\sqrt{r^2+z^2}}}{\sqrt{r^2+z^2}}\right]; \qquad (38)$$

This scalar potential function ϕ has no harmonic phenomenon.

4. Derivation and Discussion of Some Important Formulae.

Based on the calculations of potential vector II and potential scalar ϕ, derived from the last section, we can have quite a detailed discussion on the following important topics:

1. The expansion of $\left(k_2^2 + \frac{\partial^2}{\partial z^2}\right)[e^{jk_2\sqrt{r^2+z^2}} / \sqrt{r^2+z^2}] \equiv T(r, z)$

Expanding the operational differential of T(r, z), we then have

$$T(r,z) = \frac{e^{ik_2R}}{R^2}\left[\frac{k_2^2 r^2}{R^2} + i\frac{k_2}{R}\left(1-\frac{3z^2}{R^2}\right)+\frac{1}{R^2}\left(\frac{3z^2}{R^2}-1\right)\right]; \qquad (39)$$

where $R=\sqrt{r^2+z^2}$. Within the radiation boundary, when $r \to \infty$, $z \to \infty$, and $R \to \infty$, only the first term in the bracket of formula (39) is still a finite quantity; that is to say,

$$T(r,z) = \frac{k_2^2 r^2}{R^3} e^{ik_2R}. \qquad (40)$$

Within the induction boundary, and especially on the surface of the antenna of a conductor, the three terms in formula (39) cannot be disregarded in our calculations.

2. Antenna radiation energy and antenna efficiency. From formula (27), induced electric field and magnetic field in free space have the following form:

$$\left.\begin{aligned} E_{2z} &= \left(k_2^2 + \frac{\partial^2}{\partial z^2}\right)\Pi - \frac{\partial}{\partial z}\phi \\ E_{2r} &= \frac{\partial^2}{\partial r\,\partial z}\Pi - \frac{\partial}{\partial r}\phi \\ H_{2\theta} &= -j\omega\varepsilon_2\frac{\partial}{\partial r}\Pi . \end{aligned}\right\} \qquad (41)$$

In the following calculation, for our later convenience, we can propose the definition of the following "Geometric Harmonic Factor" $g_0(h)$:

$$\begin{aligned} g(h) &= \frac{2k_2h+\sin 2k_2h}{2k_2^2\sin 2k_2h} \equiv \frac{1}{k_2^2}g_0(h) \\ g_0(h) &= \frac{2k_2h+\sin 2k_2h}{2\sin 2k_2h} . \end{aligned} \qquad (42)$$

Substituting vector II and scalar ϕ in formulae (37) and (38), derived from the preceeding section, into formula (41), we can obtain the different components of the electromagnetic field of the radiation boundary:

$$\left.\begin{aligned} E_{2z} &= e^{j(k_2R-\omega t)}\left(\frac{V_0\rho_0 b}{2}\right)\left[k_2^4 g(h)\frac{r^4}{R^5} - k_2^2\frac{r^2 z}{R^4}\right], \\ E_{2r} &= e^{j(k_2R-\omega t)}\left(\frac{V_0\rho_0 b}{2}\right)\left[-k_2^4 g(h)\frac{r^3 z}{R^5} - k_2^2\frac{r^3}{R^4}\right], \\ H_{2\theta} &= e^{j(k_2R-\omega t)}\left(\frac{V_0\rho_0 b}{\omega\mu_2}\right)\left[-k_2^s g(h)\frac{r^3}{R^4}\right]. \end{aligned}\right\} \qquad (43)$$

Within the radiation boundary, the average time value of energy flow vector (Wulf-Poynting vector) is

$$\mathbf{S} = \frac{1}{2}\mathbf{E}_2 \times \mathbf{H}_2^* = \frac{1}{2}\left[\hat{z}(E_{2r}H_{2\theta}^*) + \hat{r}(-E_{2z}H_{2\theta}^*) + \hat{\theta}(0)\right], \tag{44}$$

Its radial component is

$$S_r = -\frac{1}{2}E_{2z}H_{2\theta}^* = \left(\frac{V_0\rho_0 b}{2}\right)\left(\frac{1}{\omega\mu_2}\right)\left[k_2^9 g^2(h)\frac{r^7}{R^9} - k_2^7 g(h)\frac{r^5 z}{R^8}\right] \tag{45}$$

Integrating S_r over the surface of the infinite large cylinder, we get the total radial energy flow W_r,

$$W_r = \left[\int_{-\infty}^{+\infty} S_r 2\pi r\,dz\right]_{r\to\infty} = \left(\frac{V_0\rho_0 b}{2}\right)^2\left(\frac{2\pi}{\omega\mu_2}\right)k_2^9 g^2(h)\left[\int_{-\infty}^{+\infty}\frac{r^8}{R^9}dz\right]_{r\to\infty}. \tag{46}$$

The second term of formula (45) is the odd function z. Therefore, its integral is zero. This shows that the component of E_{2z}, or E_{2z} plus the second term of the formula, brought about by charges on the two bottoms of the antenna, does not actually produce radiation. This effect tells us that the charges on the two bottoms are the incontinuity of the locality of this antenna system. It merely affects the induced electric field. Performing the integraion of formula (46), we get

$$W_r = \left[\frac{(V_0 2\pi\rho_0 b)^2\omega^4\mu_2^2\varepsilon_2^3}{12\pi\sqrt{\mu_2\varepsilon_2}}\right]\left[\frac{48}{35}g_0^2(h)\right] = \left[\frac{P_0^2\omega^4\mu_2\varepsilon_2}{12\pi}\sqrt{\frac{\mu_2}{\varepsilon_2}}\right]\left[\frac{48}{35}g_0^2(h)\right]; \tag{47}$$

where $P_0 = (\varepsilon_2 V_0 2\pi\rho_0 b) = (p_0 2\pi\rho_0 b)$ represents the total dipole moments of the source of voltage of the thin plate. The first term of formula (47) expresses the energy of radiation of a dipole with finite moment P_0 per second. The second term is a dimensionless factor. It expresses the radiation efficiency of this finite cylindrical antenna compared to the radiation of a lengthless dipole.

Taking the surface integral of S_r over the two bottom surfaces of the cylinder, we get the total radial energy flow along $\pm$ z.

$$W_z = \left[\int_0^\infty (+S_z)2\pi r\,dr\right]_{z\to+\infty} + \left[\int_0^\infty (-S_z)2\pi r\,dr\right]_{z\to-\infty} \equiv 0. \tag{48}$$

This is precisely what we desire. Because of the antisymmetry of the voltage and induction current on the two half-sections of the antenna, the total radial energy flow is zero. Taking radiation, this proves that the component of the dielectric E_r of the antenna is disregarded in calculation.

We can see from the above that the total energy flow of the antenna radiation system is completely expressed by formula (47). Suppose we let

$$W_0 = \frac{P_0^2 \omega^4 \mu_2 \varepsilon_2}{12\pi} \sqrt{\frac{\mu_2}{\varepsilon_2}} \tag{49}$$

represent the radiation of a Hertz dipole P_0. We can then use η to represent the efficiency of this antenna:

(50)

$$W = W_r = \eta W_0;$$

where $$\eta = \frac{48}{35} g_0^2(h) = \frac{48}{35}\left(\frac{2k_2 h + \sin 2k_2 h}{2 \sin 2k_2 h}\right)^2.$$

(51)

When the half-length (h) of the antenna is almost equal to one-quarter the wave-length ($\lambda_0/4$) in free space, or some multiple of it, the efficiency then becomes very high. It is also this "antenna and free space" system, and the frequency of the source of external energy which has radiation harmony. Previous experience tells us that it is correct both in principle and through calculation to view the simple theorem of energy source-excited antenna as a real radiation boundary relationship. The source of energy itself does not directly radiate, but has radiation harmony with "antenna and free space".

3. The Eigen radiation resistance of center excited antenna. Before defining Eigen radiation resistance, we must first calculate the radiation component of the current in the antenna at $z=\pm 0$. The latter is the branchcut integral in the II function. From formulae (28), (37), and (39), we get the following:

$$I_r(0) = \frac{-j\omega \varepsilon_2 2\pi \rho_0}{b}\left[\frac{\partial}{\partial r}\Pi\right]_{\substack{z=0\\ r=\rho_0}} = \frac{V_0 \pi \rho_0 b}{\omega \mu_2} k_2^3 g_0(h)\, e^{j(k_2\rho_0 - \omega t)}; \tag{52}$$

Its effective value is

$$I_{r,\mathrm{eff}} = \frac{V_0 \pi \rho_0 b}{\omega \mu_2 \sqrt{2}} k_2^3 g_0(h). \tag{53}$$

The definition of Eigen radiation resistance, R_{rad}, can be determined from the following formula:

$$W = (I_{r,\mathrm{eff}})^2 R_{\mathrm{rad}}. \tag{54}$$

From formula (54), we get

$$R_{\mathrm{rad}} = \frac{32}{35\pi}\sqrt{\frac{\mu^2}{\varepsilon^2}} = \frac{32}{35\pi}(120\pi) = 109.7; \tag{55}$$

It is almost one-quarter the radiation resistance of free space to normal electro-magnetic wave. From its definition, one can see that it is really an Eigen resistance with center-excited antenna, and has no relationship to the length of the antenna.

4. The radiation resistance of center-excited antenna excitation point. The definition of excitation point radiation resistance is

$$I_{r,\mathrm{eff}} R_{\mathrm{ant}} = V_0. \tag{56}$$

Substituting $I_{r,\mathrm{eff}}$ of formula (53) into the above formula, we get

$$R_{\mathrm{ant}} = \frac{\sqrt{2}}{4\pi^3}\frac{1}{g_0(h)}\frac{\lambda_0^2}{\rho_0 b}\sqrt{\frac{\mu_2}{\varepsilon^2}}. \tag{57}$$

We see from this that it varies as the two following factors vary: geometric harmonic factor $g_0(h)$ and wave length-sectional factor $\lambda_0^2/\rho_0 b$. Since the current in the antenna varies according to this resistance, it is the radiation resistance of cylindrical antenna excitation voltage, and the free space system. The two geometrical factors above also reveal the importance of the geometric structure of this antenna system. The higher frequency the external energy source excites, or the shorter the wave-length, the smaller will be this resistance, and the more effective a radiator will be this antenna. If the frequency of external excitation is stationary, then under geometric harmonic conditions ($g_0(h) \to \infty$), the resistance is the smallest and the radiation component of the current is the greatest. As far as the structure of a cylindrical antenna is concerned, this relationship is naturally obvious.

5. Distribution of the current radiation component on the antenna. From the first term of the II function expansion of the current radiation component, the distributed portion on the antenna has the following form:

$$I_r(z) = \frac{V_0 4\pi^3 b\, g_0(h)}{\zeta_0 \lambda_0^2}\cdot\frac{\rho_0^2}{(\rho_0^2+z^2)^2}\cos\left(k_2\sqrt{\rho_0^2+z^2}\right), \quad \zeta_0 = \sqrt{\frac{\mu_2}{\varepsilon_0}}. \tag{58}$$

Just as we desired, it is equal to the external voltage V_0; however, its distribution has a very large difference from a real cosine curve distribution.

6. Distribution of total electric current on the antenna. Including the total current of a radiation component and an induction component, its distribution can be obtained from the following equation:

$$I(z) = e^{-j\omega t}\left(\frac{V_0 \pi \rho_0^2 b}{j\omega \mu_2}\right) g_0(h) \left[\frac{\partial}{\partial r} T(r, z)\right]_{r=\rho_0}; \tag{59}$$

where T(r, z) is expressed by formula (39), and

$$\frac{\partial}{\partial r} T(r, z) = e^{jk_2 R}\left[\frac{jk_2^3 r^3}{R^4} + \frac{4k_2^2 z^2 r - 2k_2^2 r^3 + 3r}{R^5} + \frac{12\, jk_2 z^2 r - 3\, jk_2 r^3}{R^6} - \frac{15 z^2 r}{R^7}\right]. \tag{60}$$

where the radiation current component as expressed by formula (58) is calculated from the first term in formula (60). The other three terms are the induction components of the current. At the proximity region of its antenna -- especially on the antenna boundary -- it is very important. From this, we get the distribution equation of the current induction component:

$$I_i(z) = \frac{V_0 \pi \rho_0^2 b}{\omega \mu_2} g_0(h)\left[\frac{4k_2^2 z \rho_0 - 2k_2^2 \rho_0^3 + 3\rho_0}{R^5} + \frac{12\, jk_2 z^2 \rho_0 - 3\, jk_2 \rho_0^3}{R^6} - \frac{15 z^2 \rho_0}{R^7}\right] \cos\left(k_2\sqrt{\rho_0^2 + z^2} - \frac{\pi}{2}\right). \tag{61}$$

At the mid-point of the antenna at z=0, the ratio of current induction component to the radiation component is

$$\frac{I_i(0)}{I_r(0)} = \frac{1}{(k_2 \rho_0)^3}\sqrt{9 - 3(k_2\rho_0)^2 + 4(k_2\rho_0)^4}\; e^{+j\varphi}; \tag{62}$$

where

$$\varphi = -\left[\pi - \arctan\left(\frac{3 - 2k_2^2\rho_0^2}{3k_2\rho_0}\right)\right] \doteqdot -\frac{\pi}{2}. \tag{63}$$

The reason that the phase angle φ is not exactly equal to $-\frac{\pi}{2}$ is because the method of approximation is used in the branchcut integral calculation of the II function. Formula (62) tells us two important things: first, when

$k_2 \rho_0 \ll 1$, as in ordinary cases, the value of this ratio is very large. That is to say, the radiation field seems to be overhead of a very large induction field having a medium, and then leaves the latter and radiates out to free space with the velocity of light c. In all current measurements along the antenna, in fact, we merely measure out the induction current component since that very small phase radiation component, or energy component, is completely overpowered by the large induction component. Second, in order to increase the current radiation component, or to get out of the antenna's radiation field, the only way is to increase the cross-sectional area or the radius of the cylindrical antenna.

7. The effects of impedance of an antenna. From the antenna model in Diagram 1, it can be seen that the radiation boundary of every thin layer of current $I(\zeta)$ of the conductor is the same. Suppose the change of energy source in the radial direction is in accord with the well-known classical formula of current and voltage distribution in cylindrical conductors.

$$V_0 \longrightarrow V_0 J_0 (\rho_0 \sqrt{k_1^2-\lambda_1^2}) \Big/ J_0 (a \sqrt{k_1^2-\lambda_1^2}), \tag{64}$$

Then, according to the principle of superposition, we need only substitute $V_0 b$, in formula (11), $I(\zeta)b$, in formula (12), and $I(+h)b$, in formula (14), into the following formulae:

$$(11)\ 中 \quad V_0 b \longrightarrow V_0 J_0 (\rho_0 \sqrt{k_1^2-\lambda_1^2})\, d\rho_0 \Big/ J_0 (a \sqrt{k_1^2-\lambda_1^2}), \tag{65}$$

$$(12)\ 中 \quad I(\zeta) b \longrightarrow I_0(\zeta)\, J_0 (\rho_0 \sqrt{k_1^2-\lambda_1^2})\, d\rho_0 \Big/ J_0 (a \sqrt{k_1^2-\lambda_1^2}), \tag{66}$$

$$(14)\ 中 \quad I(+h) b \longrightarrow I_0(+h)\, J_0 (\rho_0 \sqrt{k_1^2-\lambda_1^2})\, d\rho_0 \Big/ J_0 (a \sqrt{k_1^2-\lambda_1^2}); \tag{67}$$

The aforementioned radiation boundary relationships are still fulfilled. In the three formulae above,

$I_0(\zeta)$ = the current volume density on the conductor surface;

$k_1^2 = \omega^2 \mu_1 \varepsilon_1 + j\omega\mu_1\sigma_1$, in the antenna conductor;

$\lambda_1 \doteq k_2$, refers to the main propagational waves in a cylindrical conductor.

The potential functions Π_0, Π, and ϕ, resulting from formulae (17), (20), and (22), all have, in the sense of ρ_0, an extra integral from $\rho_0 = 0$ to $\rho_0 = a$. The other derivation and generalization are completely unaffected.

When a cylindrical antenna is supplied energy by a pair of transmission lines of the same quality and radius, the above transformation and integration will describe very accurately the effects of impedance in an antenna. Detailed calculation is very direct, and it is simplified here.

Assume that in the middle of the antenna we use other methods of supplying energy then at $z=\pm 0$, or at the point of input, the distributed function of voltage and current on the radius must be determined or described beforehand. Only then can we consider calculation of the effects and results of impedance in the antenna.

Appendix

Derivation of Hertz's differential function formula (11) of energy source with voltage V_o.

Vector potential A of d'Alembert's equation

$$\nabla^2 A - \mu\varepsilon \frac{\partial^2}{\partial t^2} A = -\mu J. \tag{A1}$$

Using reasonable M.K.S. units to calculate, the solution is

$$dA = \frac{\mu J(\zeta)\, d\zeta\, dS}{4\pi R}\, e^{j(k_2 R - \omega t)}; \tag{A2}$$

where $J(\zeta)$ is along the ζ-current volume density direction, R is the distance from observation point to volume element $J(\zeta)d\ dS$. The equivalent definitive formula of the Hertz II_o function is

$$dA = \mu\varepsilon \frac{\partial}{\partial t}(d\Pi_0) = -j\omega\mu_2\varepsilon_2\, d\Pi_0. \tag{A3}$$

Combining formulae (A2) and (A3), we obtain

$$d\Pi_0 = \frac{J(\zeta)\, d\zeta\, dS}{-j\,4\pi\omega\varepsilon R}\, e^{j(kR-\omega t)}. \tag{A4}$$

The current $J(\zeta)$ on length $d\zeta$ expresses the opposite charges on the two bottom surfaces, their relationships is

$$J(\zeta) = \frac{\partial}{\partial t} q = -j\omega\, q(\zeta). \tag{A5}$$

Two opposite charges $\pm qdS$, a distance of $d\zeta$ apart, equivalent to a capacitance system, and possessing capacitance

$$dC = \frac{\varepsilon\, dS}{d\zeta}. \tag{A6}$$

Dividing dC by qdS, we get $d\zeta$ the two terminals, the voltage difference V_o:

$$V_0 = \frac{q\,dS}{dC} = \frac{q\,d\zeta}{\varepsilon} = \frac{p_0}{\varepsilon} = \frac{J(\zeta)\,d\zeta}{-j\omega\varepsilon}; \tag{A7}$$

where $\rho_o = qd\zeta$ is the unit area's dipole moment. Substituting formula (A7) into formula (A4), we get

$$d\Pi_0 = \hat{s}\,\frac{V_0\,dS}{4\pi R}\,e^{j(kR-\omega t)}. \tag{A8}$$

So, in a small space $d\zeta$, the two terminals have the energy source of the voltage V_o, equivalent to a similar small space with a classical Hertz dipole. In the preceeding part of this thesis, we have used this formula to calculate the expansion form of the energy source field.

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